RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022

FIRST YEAR [BATCH 2021-23]

Computer Science & Machine Intelligence

Paper : II [CC2]

Date : 21/03/2022

Time : 11 am – 1 pm

Full Marks : 50

Answer **any five** questions of the following: [5×10]

1. a) Consider the random experiment of tossing two unbiased coins simultaneously and coming up with the random variable X as the number of tails. Show that the cumulative distribution function F(x) is a non-decreasing and right continuous function.

   b) From the function F(x) in the example above, come up with the probability mass function of X. [5+5]

2. Suppose, on an average, 'm' customers arrive at a service station during the time interval [10 AM – 11 AM]. Define the random variable T as the waiting time before the first customer arrives. Find the probability density function of the random variable T. [Assume that the number of customers arriving at the service station during the above-mentioned interval follows Poisson distribution with parameter 'm'.] [10]

3. Consider the following transition probability matrix concerning the two-state Markov chain about the weather conditions - '0' if it rains and '1' if does not :

$$\begin{vmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{vmatrix}$$

   Find the probability that it'll rain five days from today given that it's raining today. [10]

4. Keeping in mind the set-up of Bernoullian trials, define the random variable Y as the trial at which the r^{th} success occurs. Come up with the probability mass function f(x) for the random variable X = (Y-r) and show that $\sum_x f(x) = 1$ [10]

5. a) Suppose a computer program has an error in it. To detect the error, the programmer conducts a series of independent tests. The probability of a random test detecting an error is 0.10. Find the expected number of tests required to detect the error.

   b) Assuming that the coins are all unbiased, in how many cases you will expect to get more than 60 heads in 100 tosses out of 10,000 cases? [Assume central limit theorem & also note $\Phi(2.5)$ = 0.9938] [5+5]

6. a) Suppose we draw a random sample $X_1, X_2, \ldots, X_n$ from the population N (μ, σ^2). Carry out, theoretically, at α% level of significance,the test for the null hypothesis $H_0: \mu = \mu_0$ against the null hypothesis $H_1 : \mu > \mu_0$ [Assume that σ^2 is unknown].

   b) Consider the regression model $y = \alpha + \beta x + \varepsilon$, where x is a non-random regressor variable and ε is the random error term. Show that OLS estimators of both α&β are unbiased. [5+5]

7. a) Find the Jacobian matrix of the function $f : R^3 \to R^2$ defined as $f(x,y,z) = (x+y+z, xyz)$ and evaluate it at (2,1,3).

b) State and prove the necessary and sufficient condition for a non-empty subset of a vector space to be a subspace of that vector space.

c) Find the change-of-basis matrix from the basis P of R^3 to another basis S of R^3, where P= {(1,2,0), (1,3,2), (0,1,3)} and S = {(2,1,4), (1,-1,2), (3,1,-2)}. [3+4+3]

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